Universitat d'Alacant

GENERALITAT VALENCIANA
Conselleria de Educación, Universidades y Empleo

Understanding the dynamics of neutron star magnetic field through 3D magneto-thermal simulations

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MAgneto-**T**hermal evolution of **I**solated **N**eutron **S**tars

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Why we need 3D magneto-thermal models?

- Understanding the variety of population of isolated neutron stars and their evolutionary paths
- Realistic magnetic topology: complex and non-axisymmetric
- The need to model cooling curves, that depend on the 3d configuration
- 3D magnetic evolution leads to the formation of hotspots on the stellar surface

[Dehman, Marino, kovlakas, Rea et al. 2024 in prep.]

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Thermal evolution & cooling curves

$$c_V(T)\frac{\partial\left(Te^{\nu}\right)}{\partial t} = \vec{\nabla}\cdot\left(e^{\nu}\hat{\kappa}\cdot\vec{\nabla}(e^{\nu}T)\right) + e^{2\nu}(Q_J - Q_{\nu})$$

Ingredients:

- Neutron star model: EoS + central pressure —> star structure & composition (fixed)
- Heat capacity $C_V(\rho, T)$: main contribution by neutrons in the core
- **Thermal conductivity** $\boldsymbol{\kappa}(\rho, T, \boldsymbol{B})$ very large (star core rapidly isothermal), dominated by electrons, becomes **anisotropic** in presence of magnetic field
- Neutrino emissivity $Q_\nu(\rho, T, \boldsymbol{B})$
- Sources of internal heat Q_j: nuclear reactions, **Ohmic dissipation**, accretion…
- Hydrostatic equilibrium models of envelope (i.e., liquid outermost 100 m), that due to its stronger gradients of density and temperature has much faster timescales than the interior
- Emission model (atmosphere, blackbody, condensed surface...)

[Potekhin et al. 2015, review]

[Pons & Viganò 2019, living review]

http://www.ioffe.fr/astro/conduct/

https://compose.obspm.fr/

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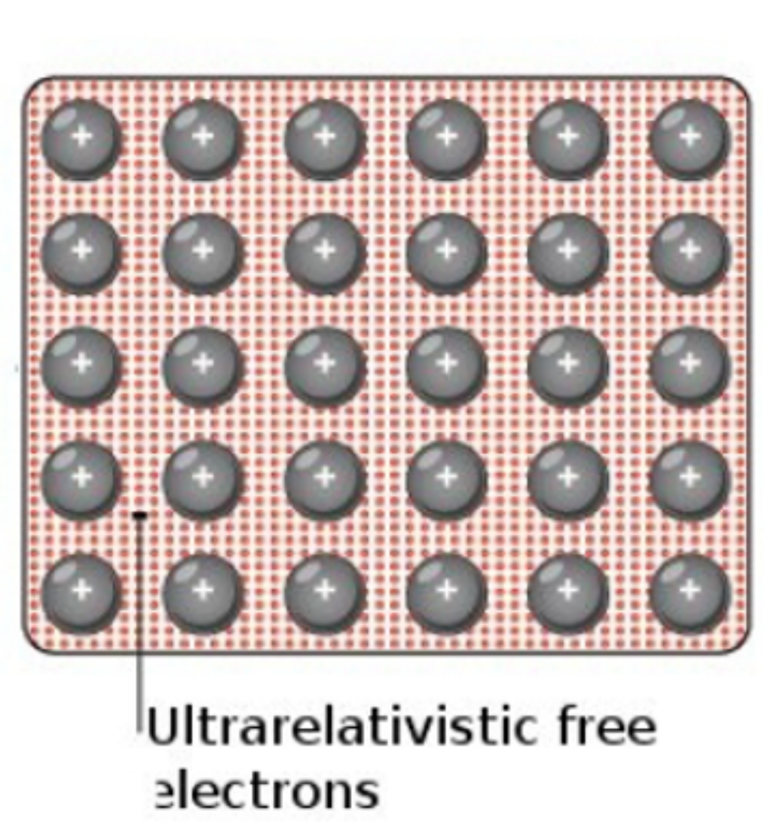
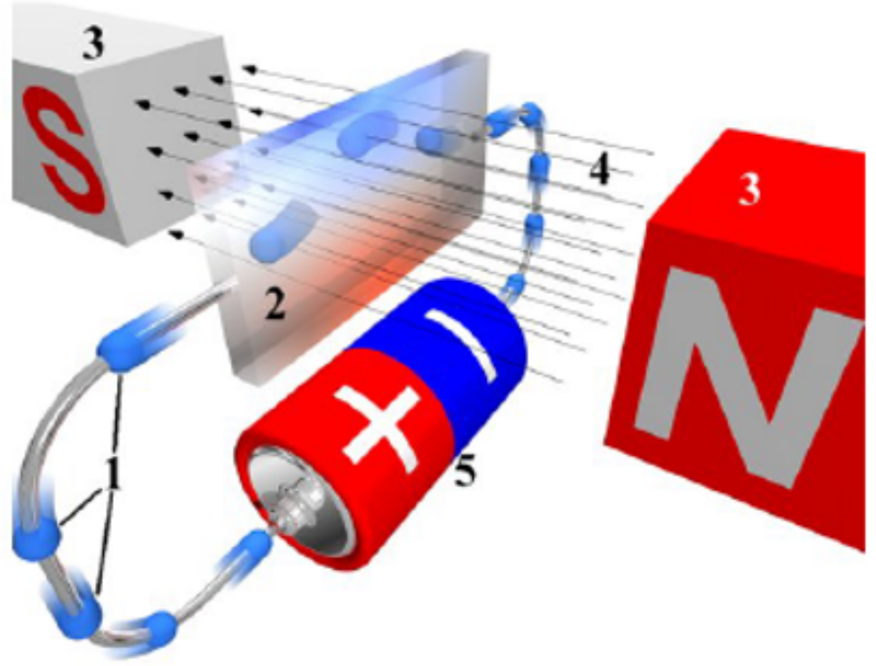
Magnetic field evolution

— Hall MHD limit —





- Neutron stars interior ⟶ complex multi-fluid system
- A solid crust is formed soon after birth ⟶ restricted nuclei mobility ⟶ conduction governed by electrons
- Core: full multi-fluid system
- Approximation: electrons MHD limit in the crust (eMHD)

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times\left[\underbrace{\frac{c^2}{4\pi\sigma_e}\nabla\times(e^{\nu}\boldsymbol{B})}+\underbrace{\frac{c}{4\pi e n_e}[\nabla\times(e^{\nu}\boldsymbol{B})]\times\boldsymbol{B}}\right]$$

(Lander's talk)

Ohmic dissipative term: the magnetic resistivity is very sensitive to temperature evolution and electron density

Hall drift term: It naturally creates magnetic discontinuity and transfers energy between different scales.

- Crustal-confined (perfect conductor at the crust-core interface).
- Potential boundary conditions (i.e. no current, $\nabla\times\boldsymbol{B}=0$) - better force-free magnetosphere.
- Divergence-free magnetic field $\nabla\cdot B=0$.

[Pons & Viganò 2019, living review]

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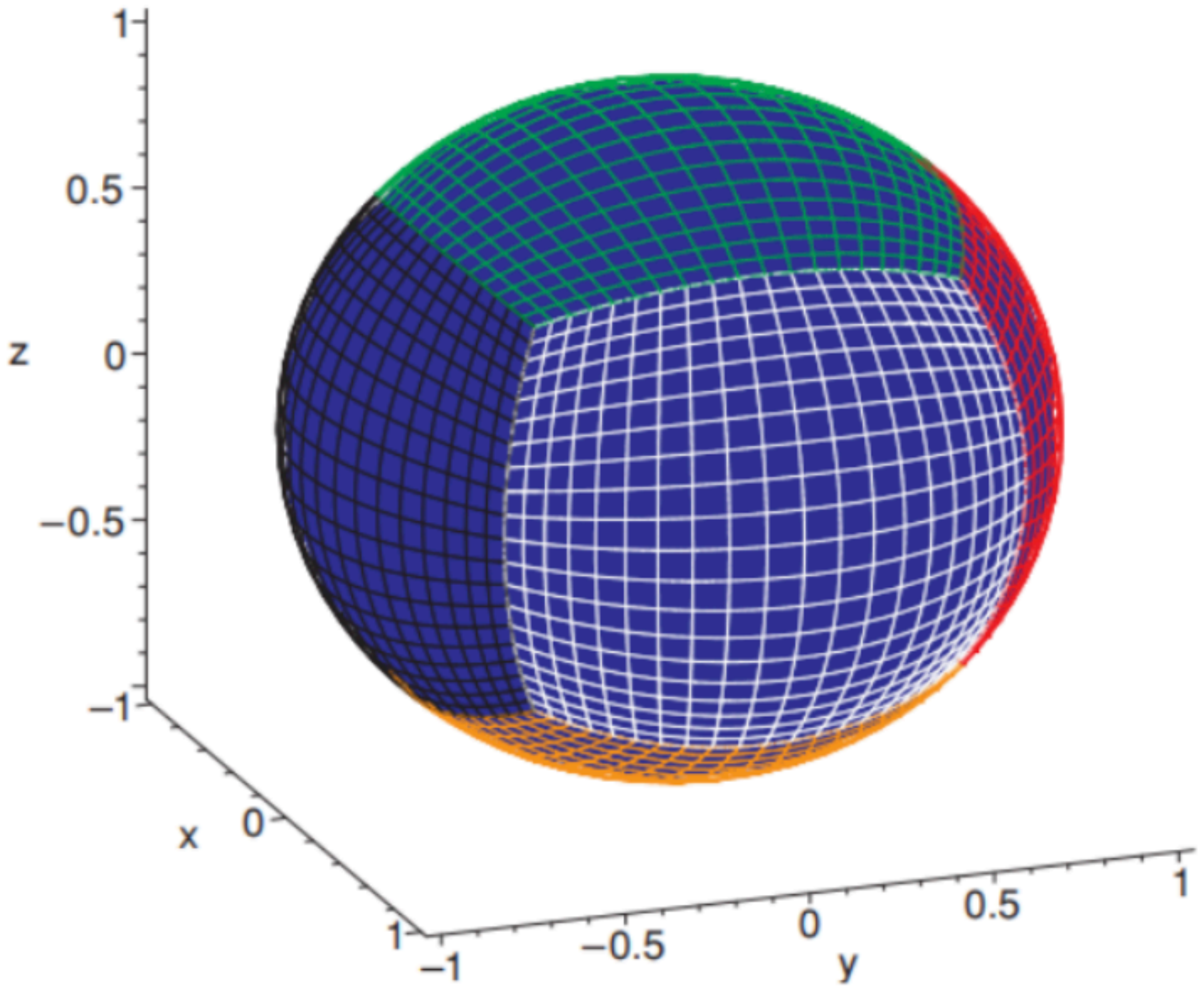
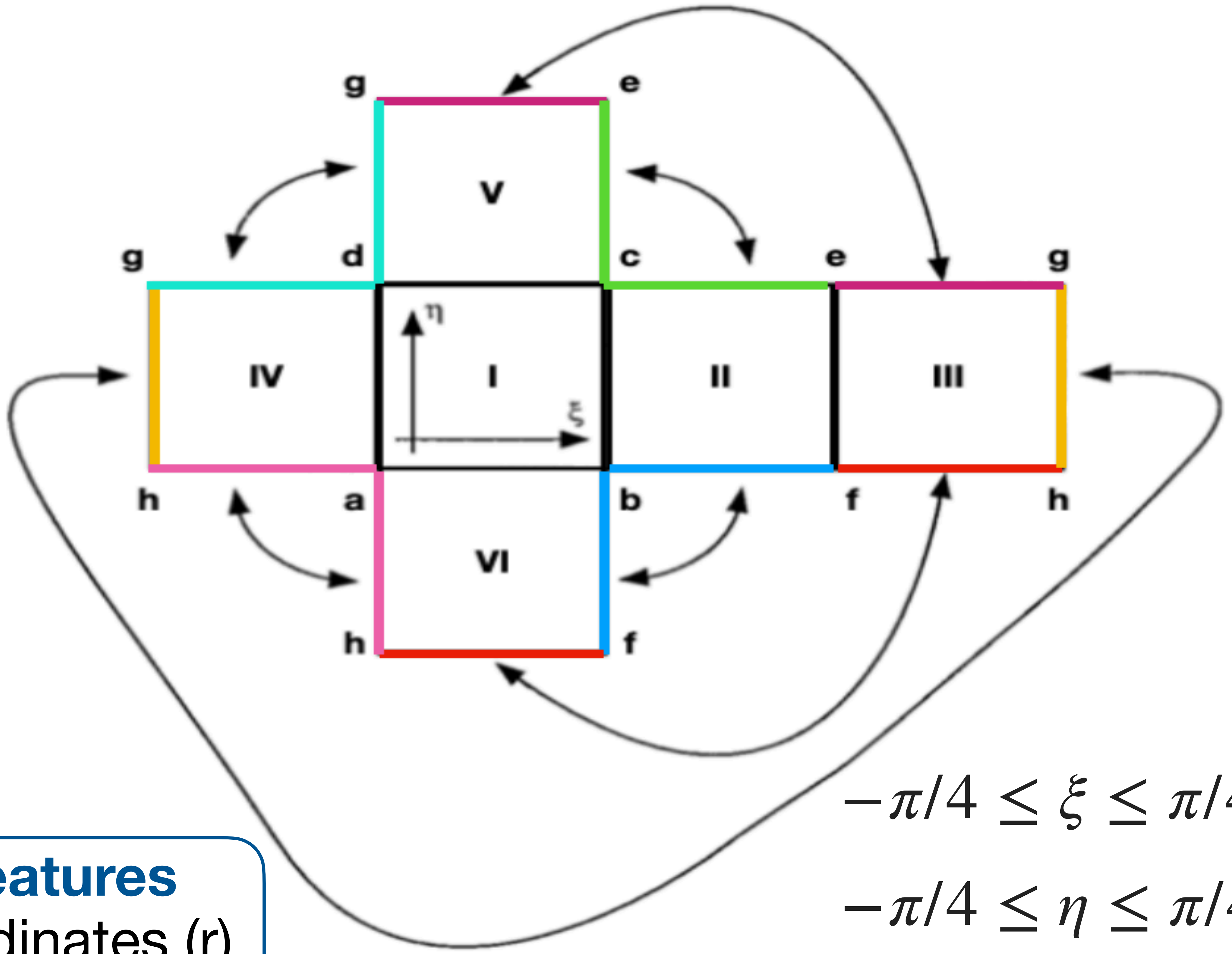
Schwarzschild cubed-sphere

In 3D spherical coordinates if you want to use **finite-volume**/difference methods, the axis is a singularity. The cubed sphere coordinates are a widely used solution, used in climate and atmospheric simulations



[Ronchi et al. 1996]



$$-\pi/4 \leq \xi \leq \pi/4$$

$$-\pi/4 \leq \eta \leq \pi/4$$

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\dfrac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} \\ 0 & -\dfrac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} & 1 \end{pmatrix}$$

non-orthogonal coordinate system

Desirable features

- Radial coordinates (r)
- No axis-singularity
- GR correction

[Dehman et al. 2022 MNRAS]

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MATINS the brand new 3D code

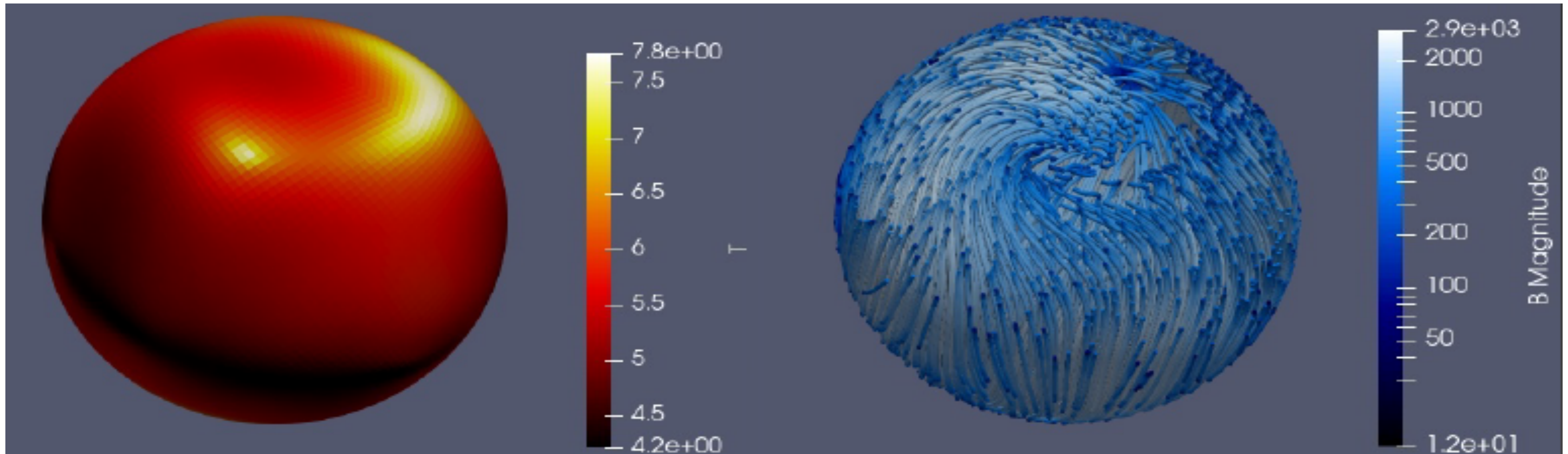
Dehman**, Viganò, Pons & Rea 2022, MNRAS* (DOI: **10.1093/mnras/stac2761**): ***Cubed-sphere grid + Magnetic formalism

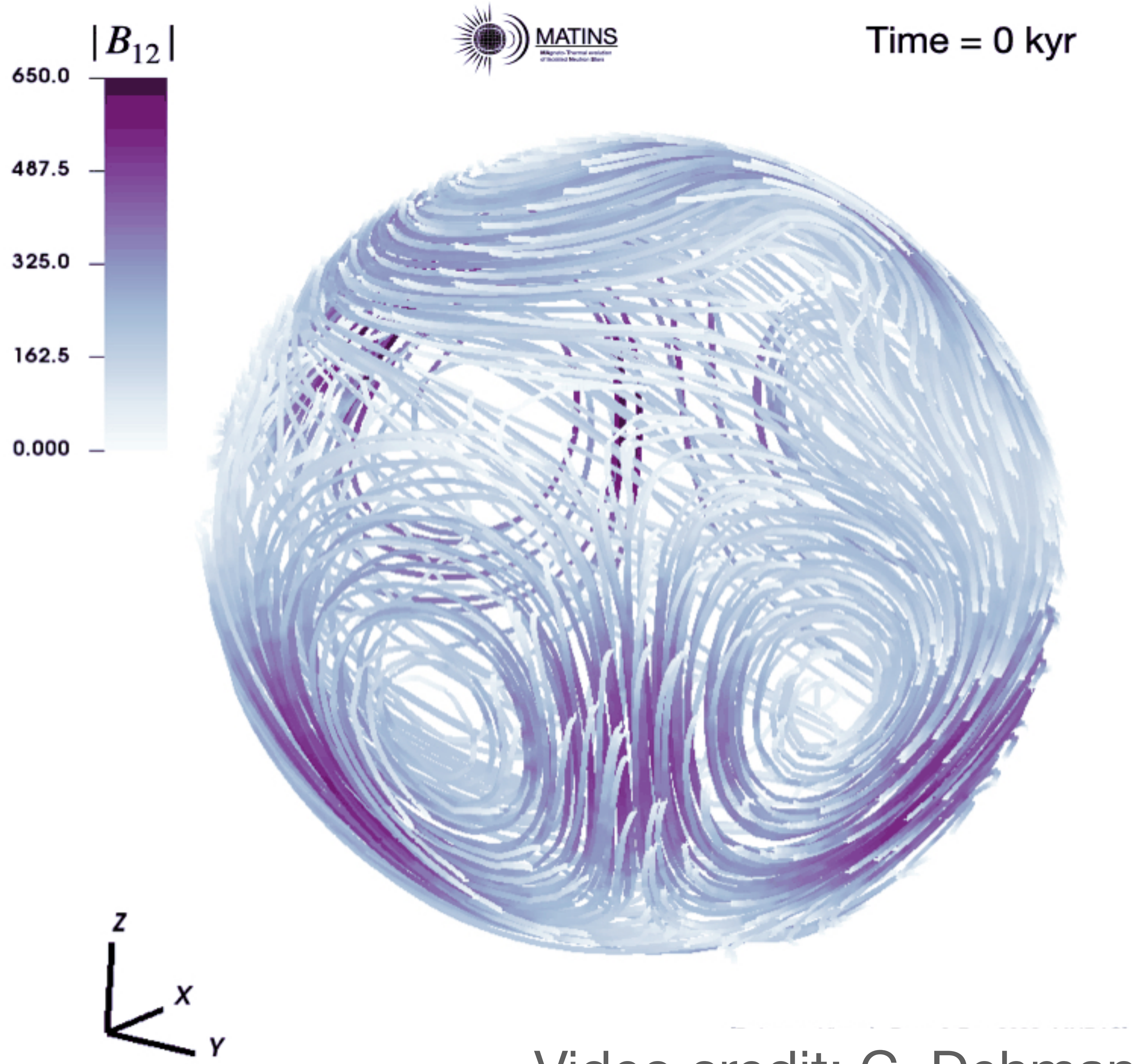
*Ascenzi, Viganò, **Dehman**, Pons & Rea, Perna 2024, submitted to MNRAS:* ***Thermal formalism*** ***(See S. Ascenzi's poster)***

Dehman**, Viganò, Ascenzi, Pons & Rea 2023, MNRAS* (DOI: 10.1093/mnras/stad1773): **First *3D magneto-thermal simulation

Soon to be public.
What's better than 2D:
- Simulation of 3D magnetic modes, hotspots, and light curves
- Better documentation, use of novel coordinates (cubed-sphere)
- Optimization and use of OpenMP

Advance obtained (only another 3D code was existing so far):
- **Realistic 3D evolution and topology, appearance of hotspots**
- **State-of-the-art microphysics and realistic structure**
- **Numerical scheme to better capture non-linear dynamics**
- **General relativistic correction**
- **State of art envelope model**
- **Flexibility in implementing new physics**
- **Documentation and modularity (for public)**





Video credit: C. Dehman

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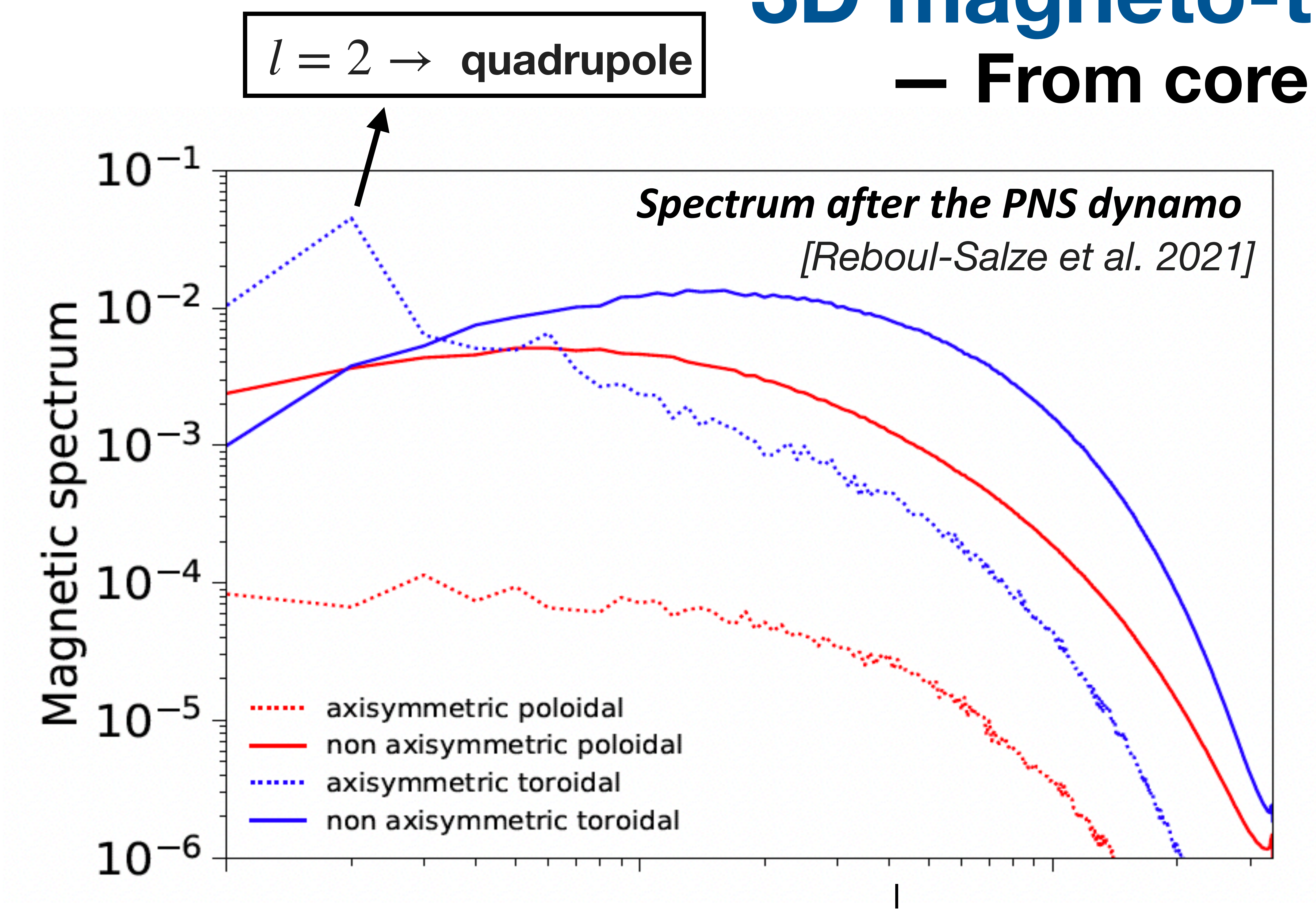
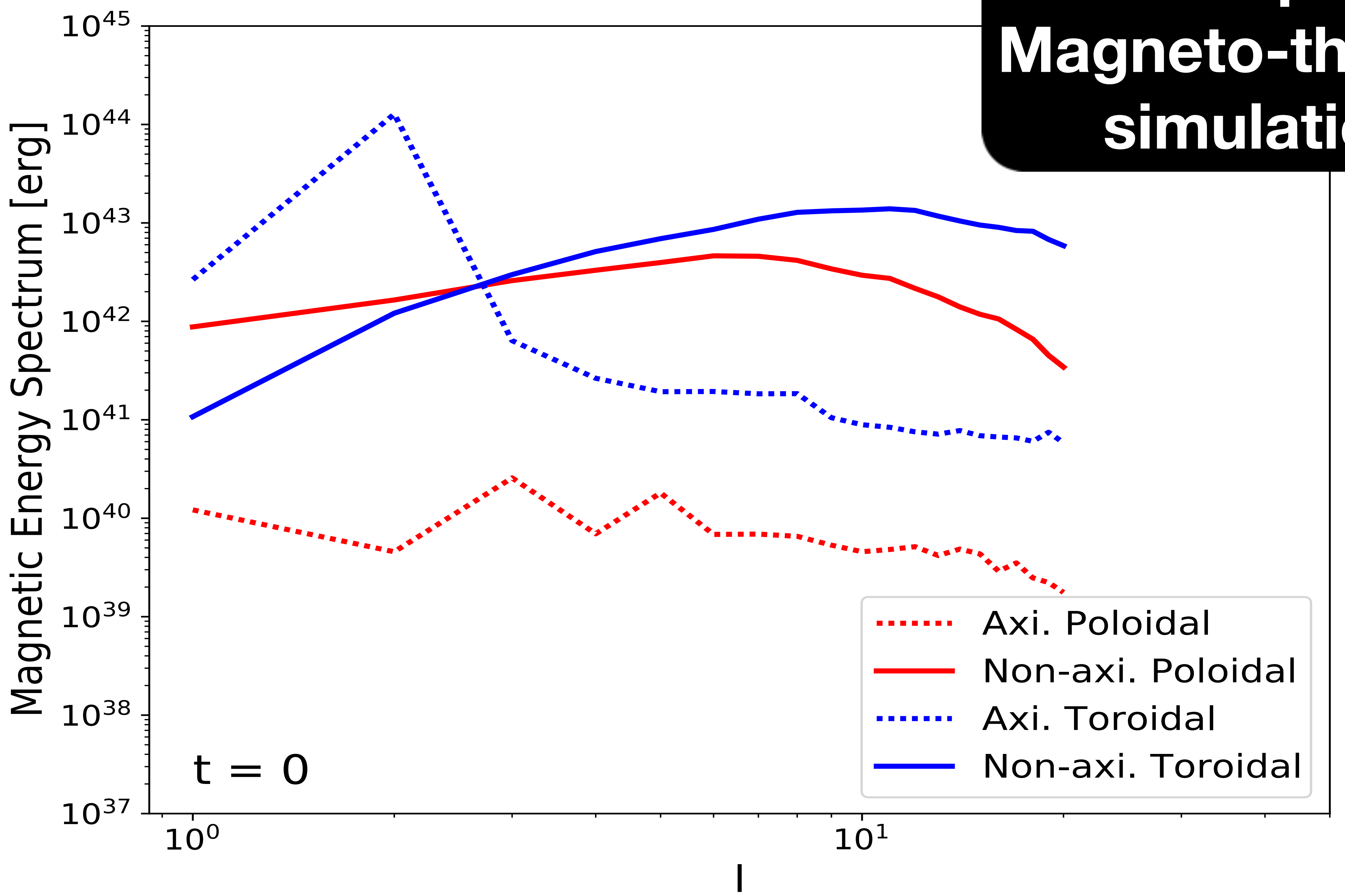
3D magneto-thermal simulations

— From core collapse dynamo —


$l = 2 \rightarrow$ **quadrupole**

Spectrum after the PNS dynamo

[Reboul-Salze et al. 2021]


First coupled 3D Magneto-thermal simulation

Magnetic energy is distributed across a broad range of scales.

Toroidal axisymmetric quadrupole and non-axisymmetric components dominate.

Dipolar field accounts for less than 5% of the total magnetic energy.

From post-collapse to neutron star phase, plenty of MHD timescales to approach an equilibrium, with dynamo still going on and at the same time dissipating the smallest scales.

But: no way to leave only a dipole or axisymmetric configurations!

$\boxed{\text{Avg}(B) \sim 10^{14}\ \text{G}} \longrightarrow$ **Magnetar-like initial field**

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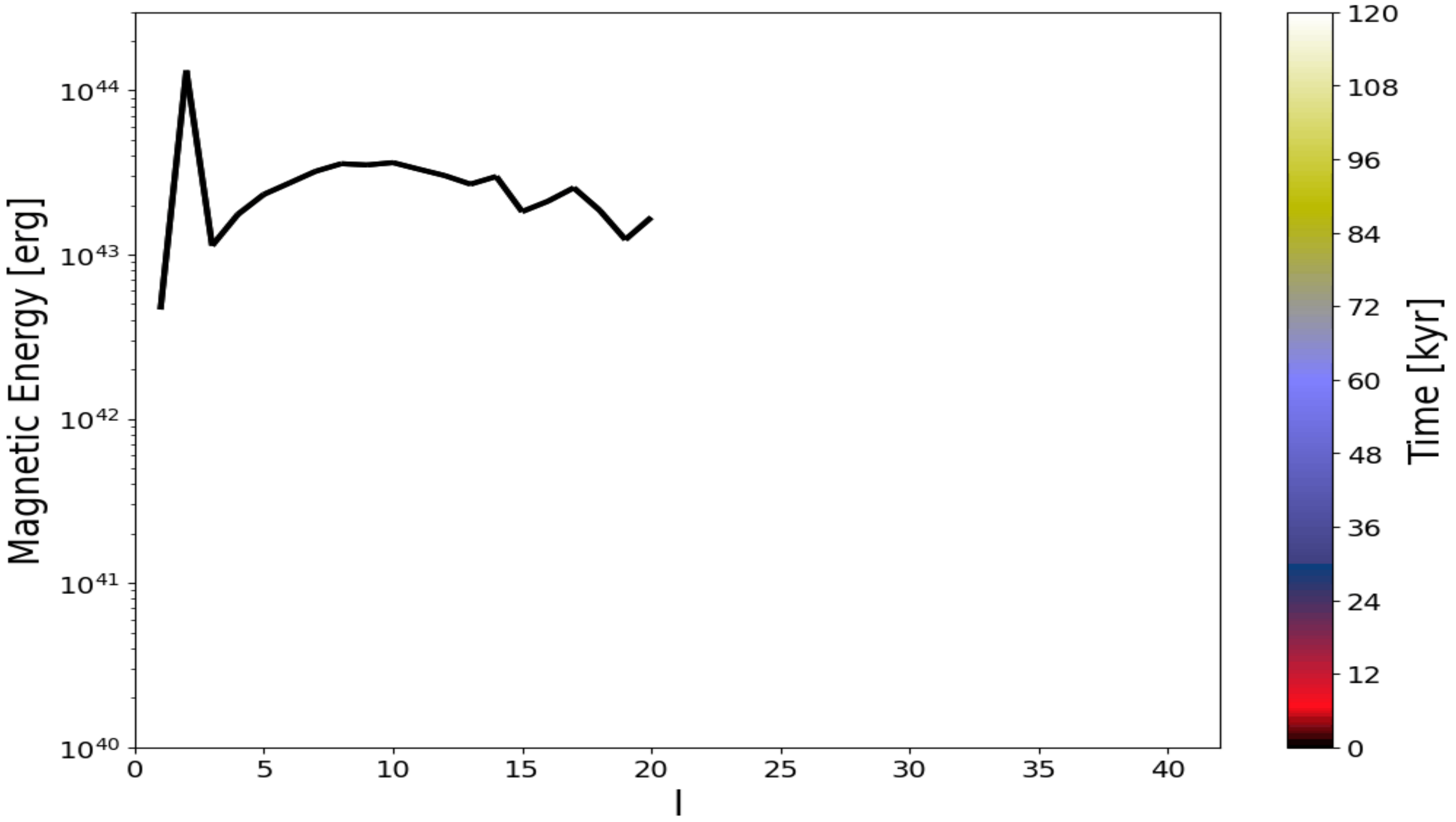
[Dehman et al. 2023b, MNRAS]

3D magneto-thermal evolution



Field keeps a strong memory of the initial large-scale structures

- **Hall Cascade:** redistribution of the magnetic energy over different spatial scales
- Small-scale multipoles dissipate faster (L^2/η_b) —> Enhanced Ohmic heating
- Hall balance is reached in the system — l^{-2} slope —
- Initial large scale quadrupole remains dominant

[Dehman et al. 2022 MNRAS]
[Dehman et al. 2023b, MNRAS]

[Goldreich & Reisenegger et al. 1992]

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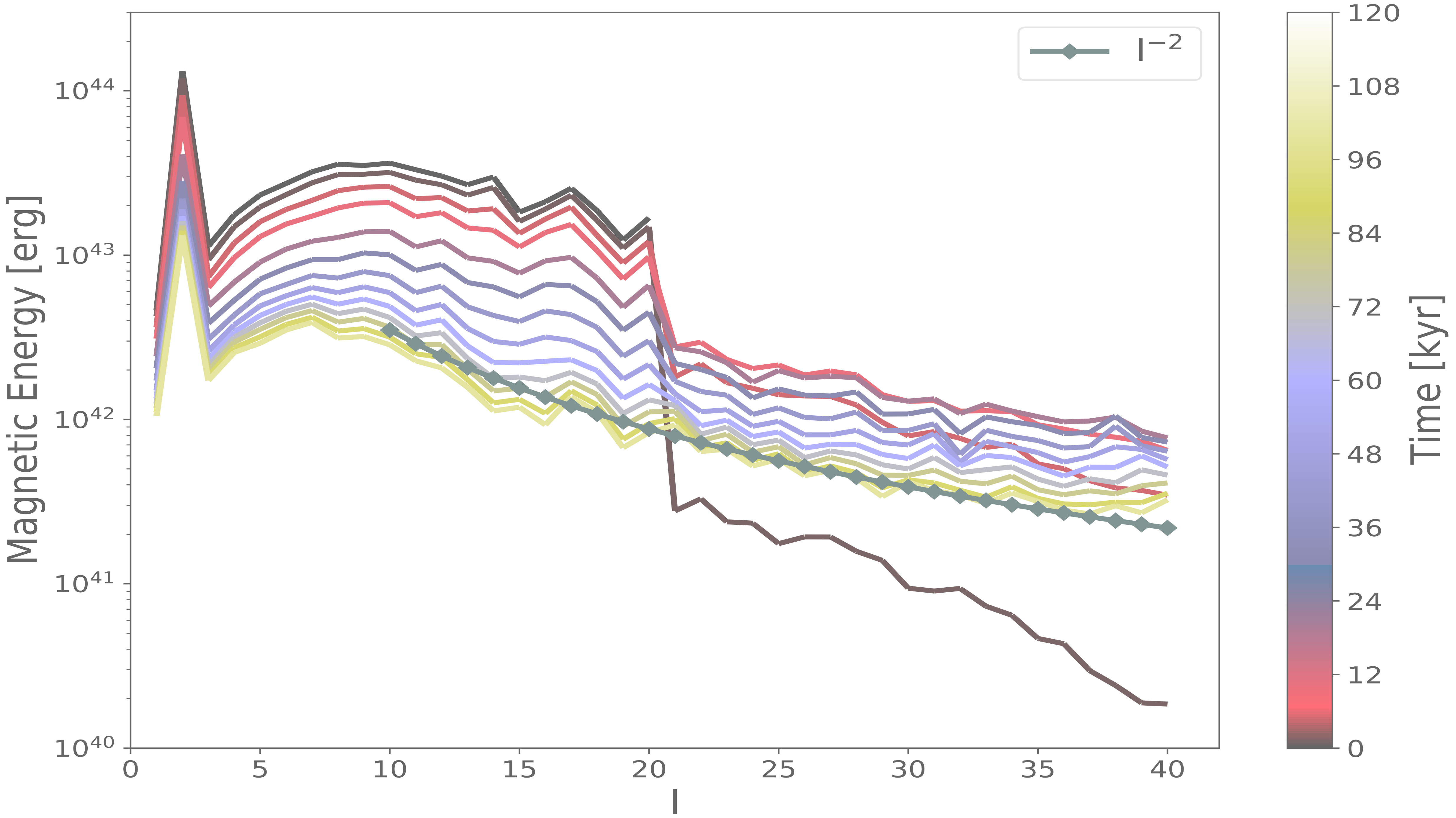
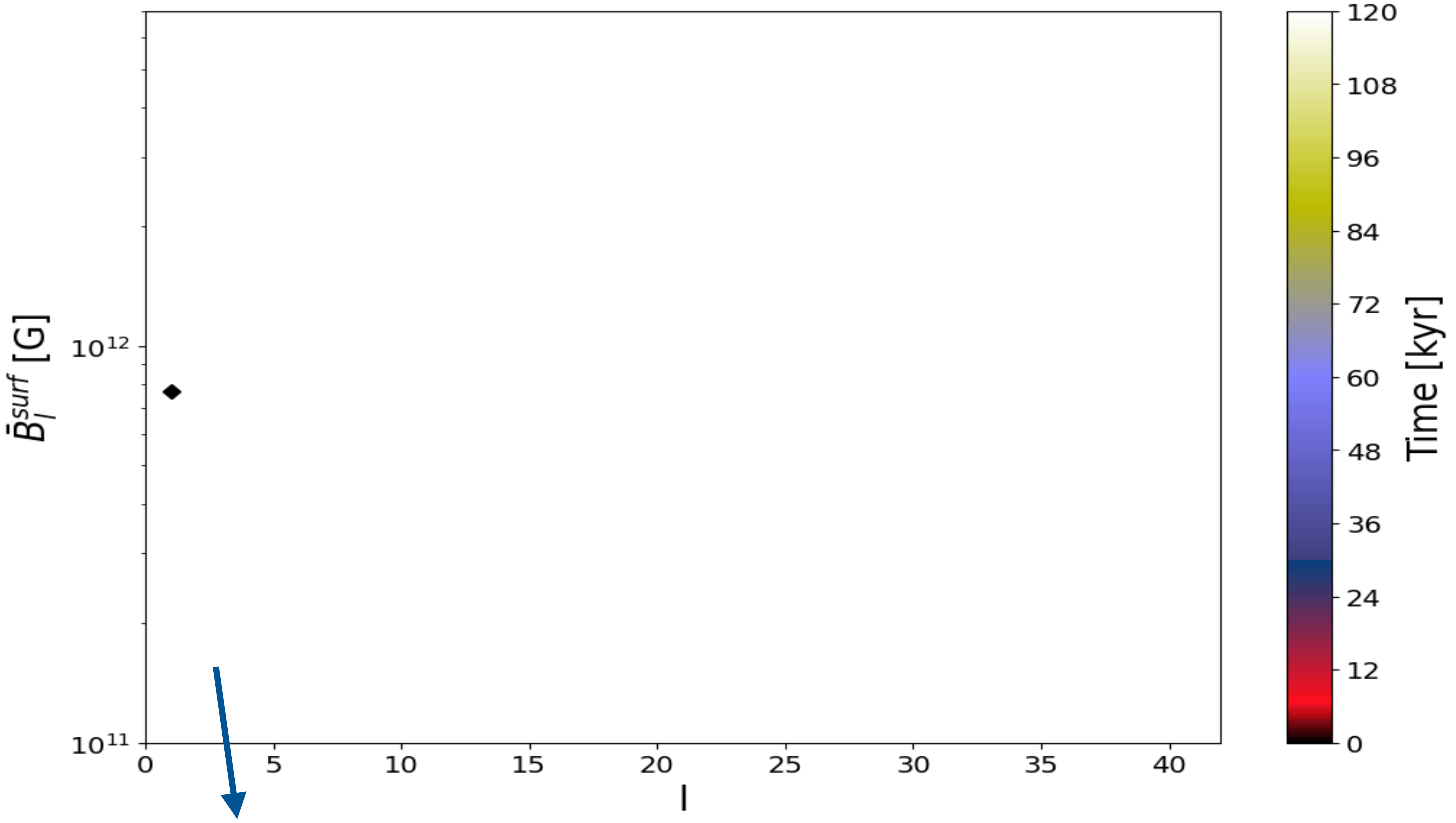
3D magneto-thermal evolution





$l = 1 \rightarrow$ **dipole**

- Initial surface dipolar field - choice of radial function

- The surface gets populated by small scale multipoles.

- The surface dipolar magnetic field does not grow in time.

[Dehman et al. 2023b, MNRAS]

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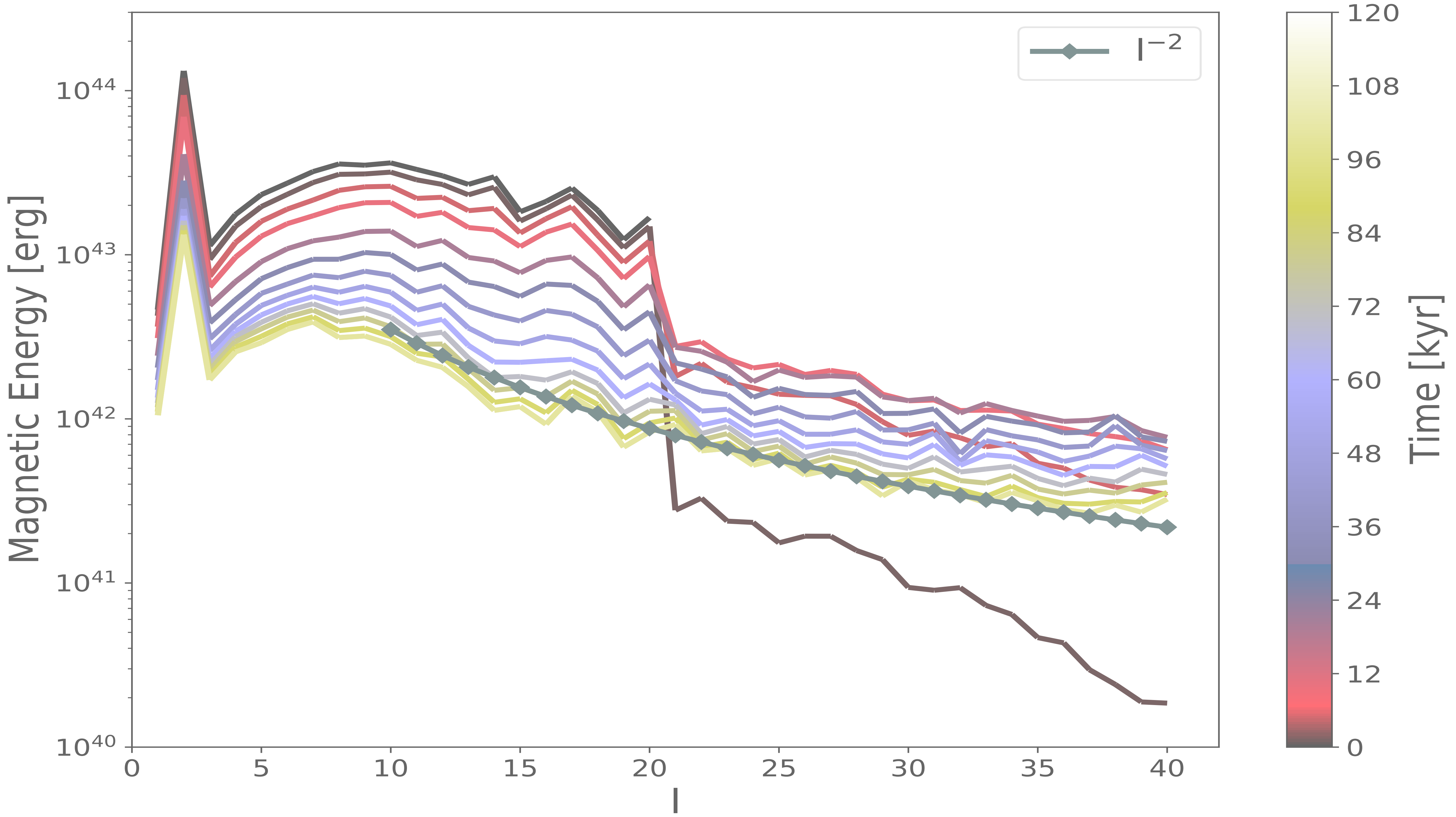
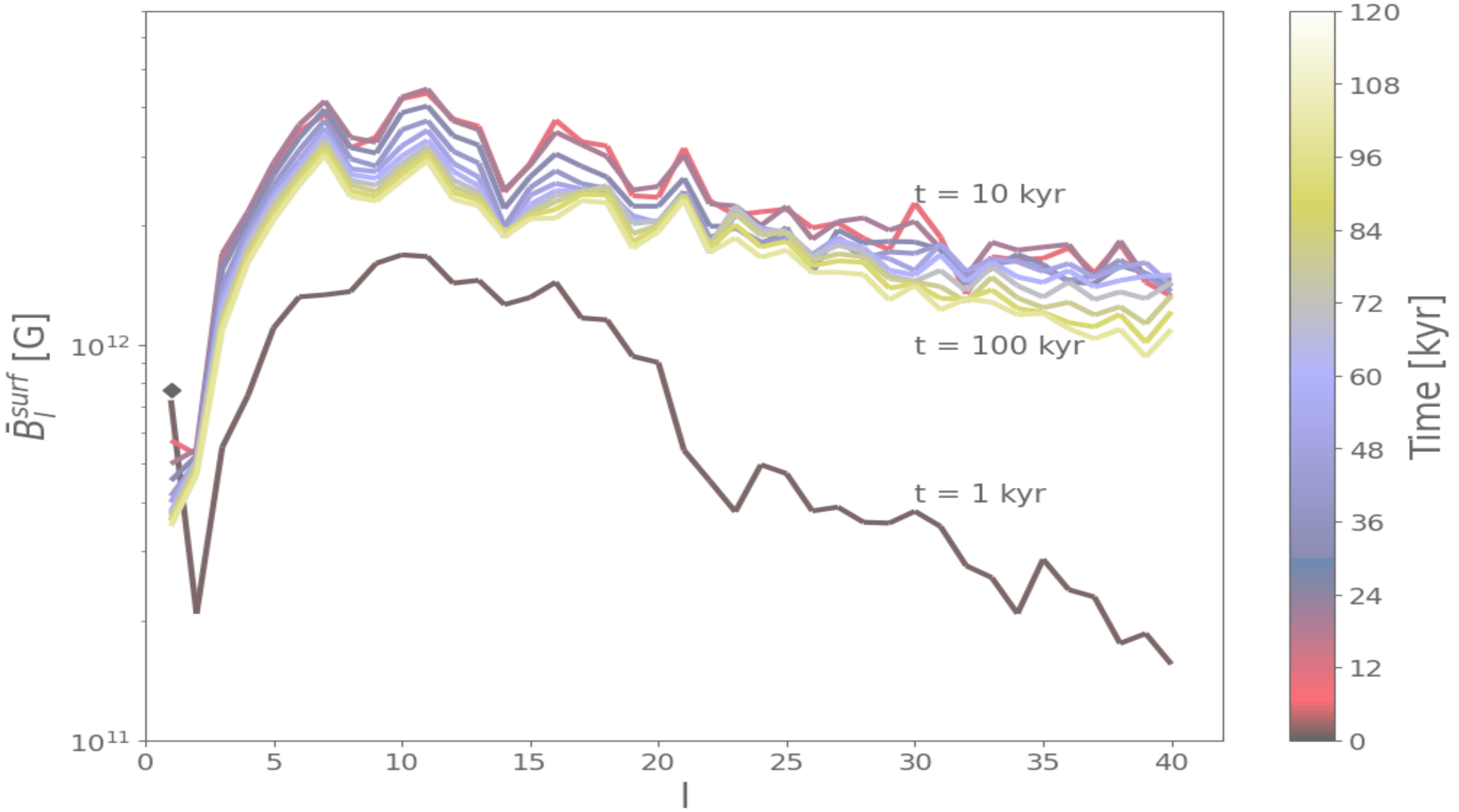
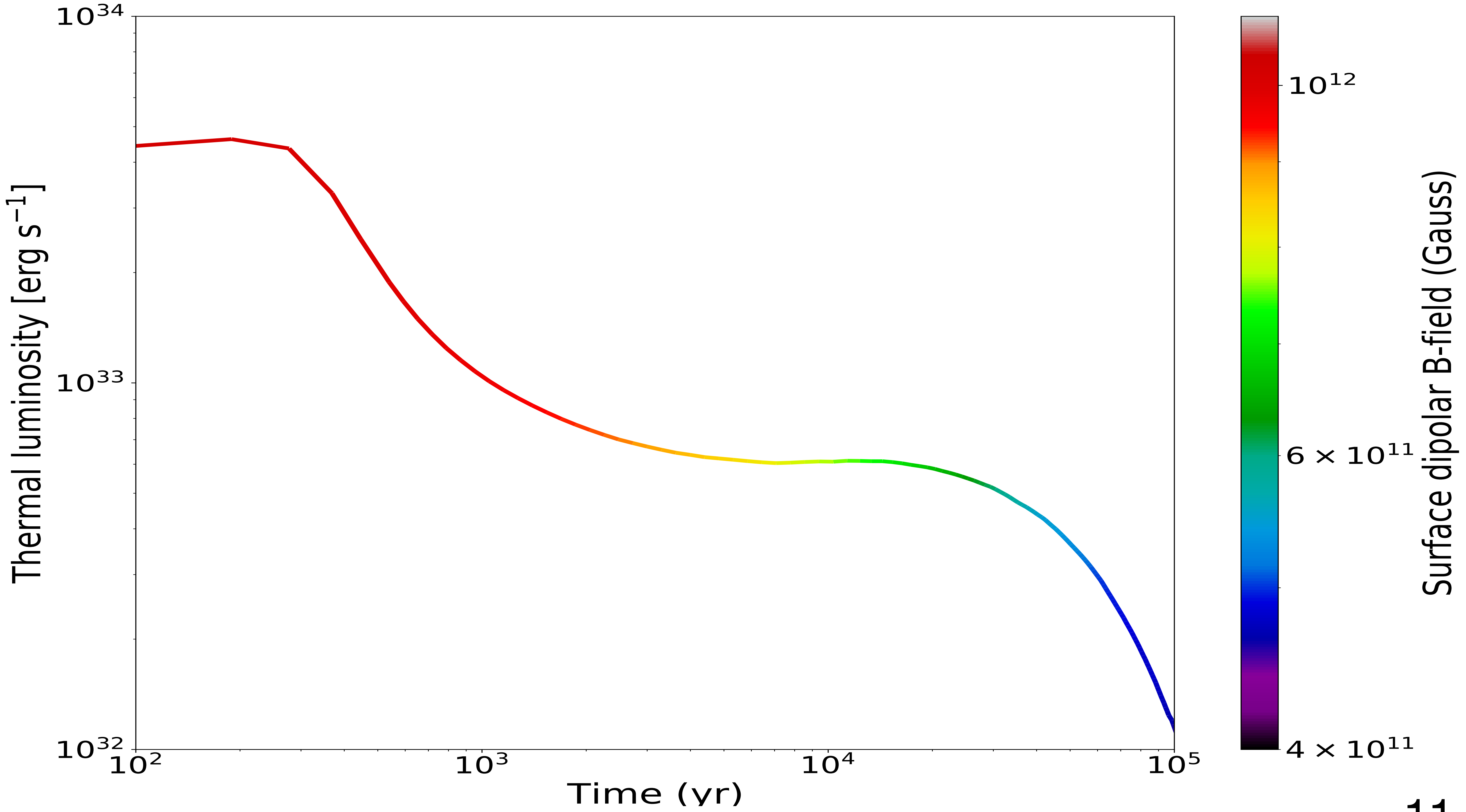
3D magneto-thermal evolution







Thermal luminosity suitable to describe CCOs & low-field magnetars.

What about Magnetars?
How can we form the strong dipolar fields, responsible for the spin-down torque?

[Dehman et al. 2023b, MNRAS]

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Reality of inverse cascade in neutron star crusts

A scenario for generating a large-scale dipolar field is through an inverse cascade, starting with an initial helical magnetic field.

“Brandenburg 2020”

Initial field:

Helical, or in other words, a force-free field.

Random initial field peaking at $l_0 \sim 100$.

Causal spectrum as used in the cosmological context.

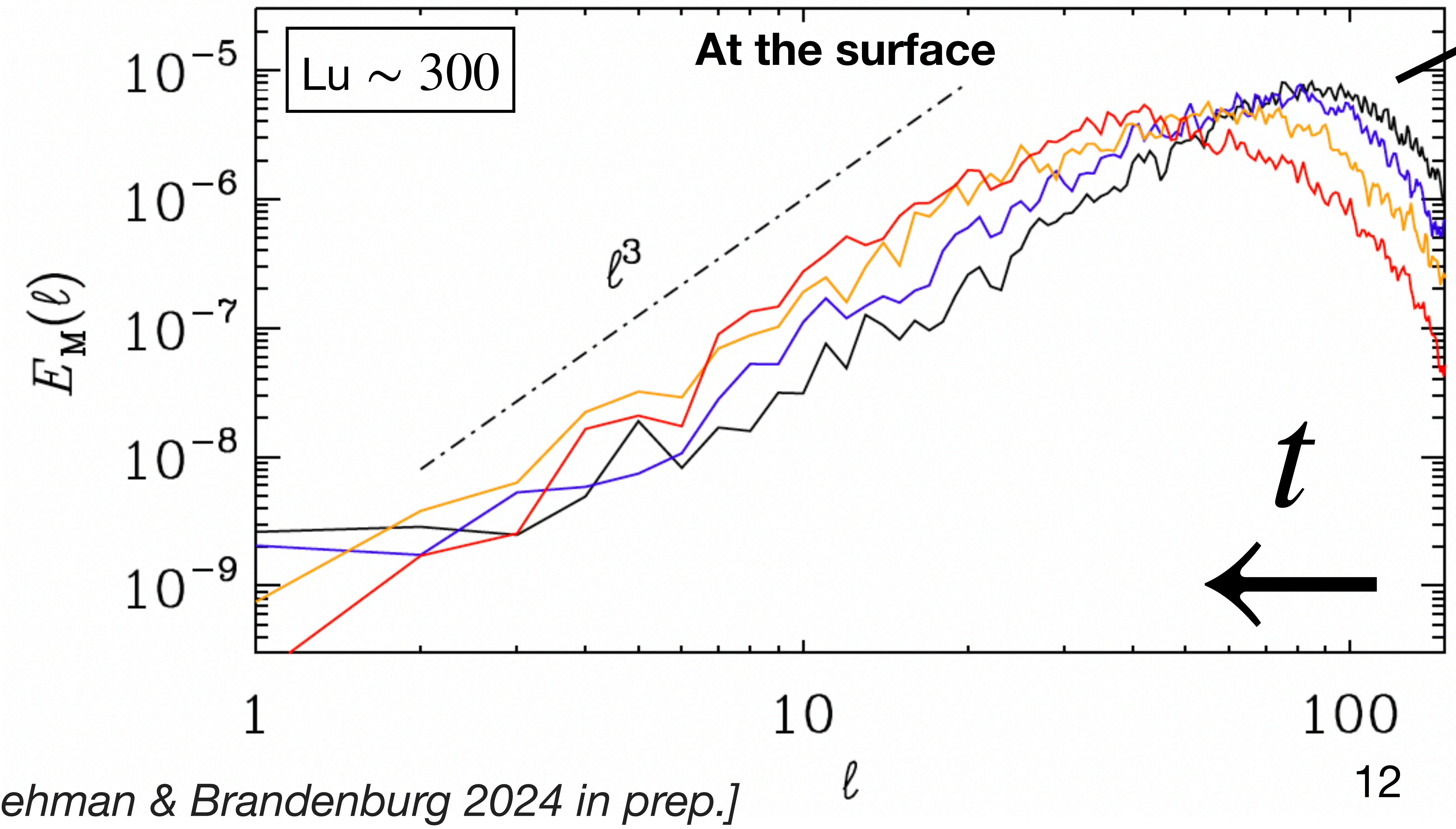
Inverse Cascade occurs!

Energy transferred from small to large-scale multipoles.

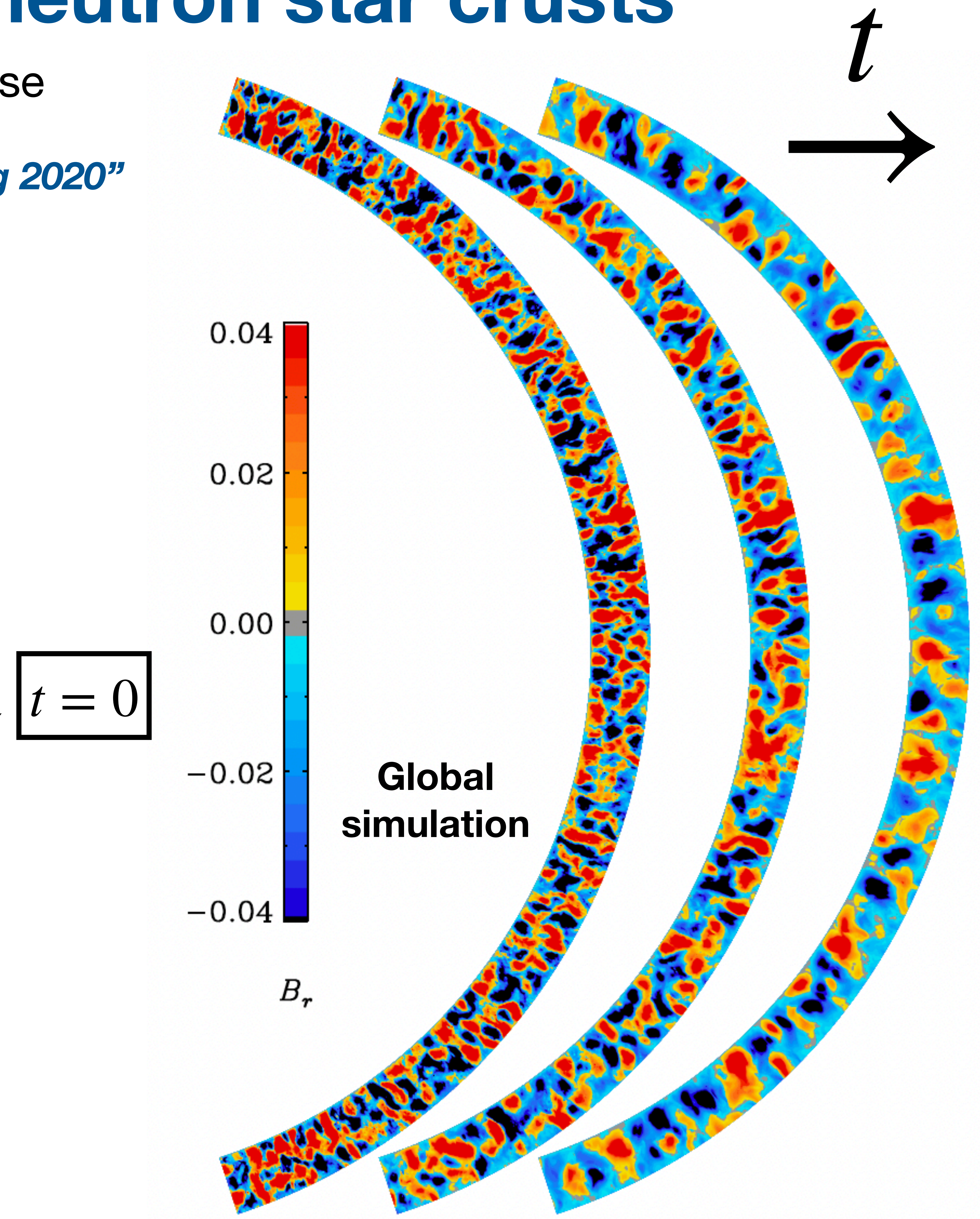
Not observed in previous neutron star simulation studies.

Magnetic field dissipates over time! **Boundary conditions' impact?**

Extreme aspect ratio —thin crust— limits the inverse cascade.



[Dehman & Brandenburg 2024 in prep.]



Pencil Code

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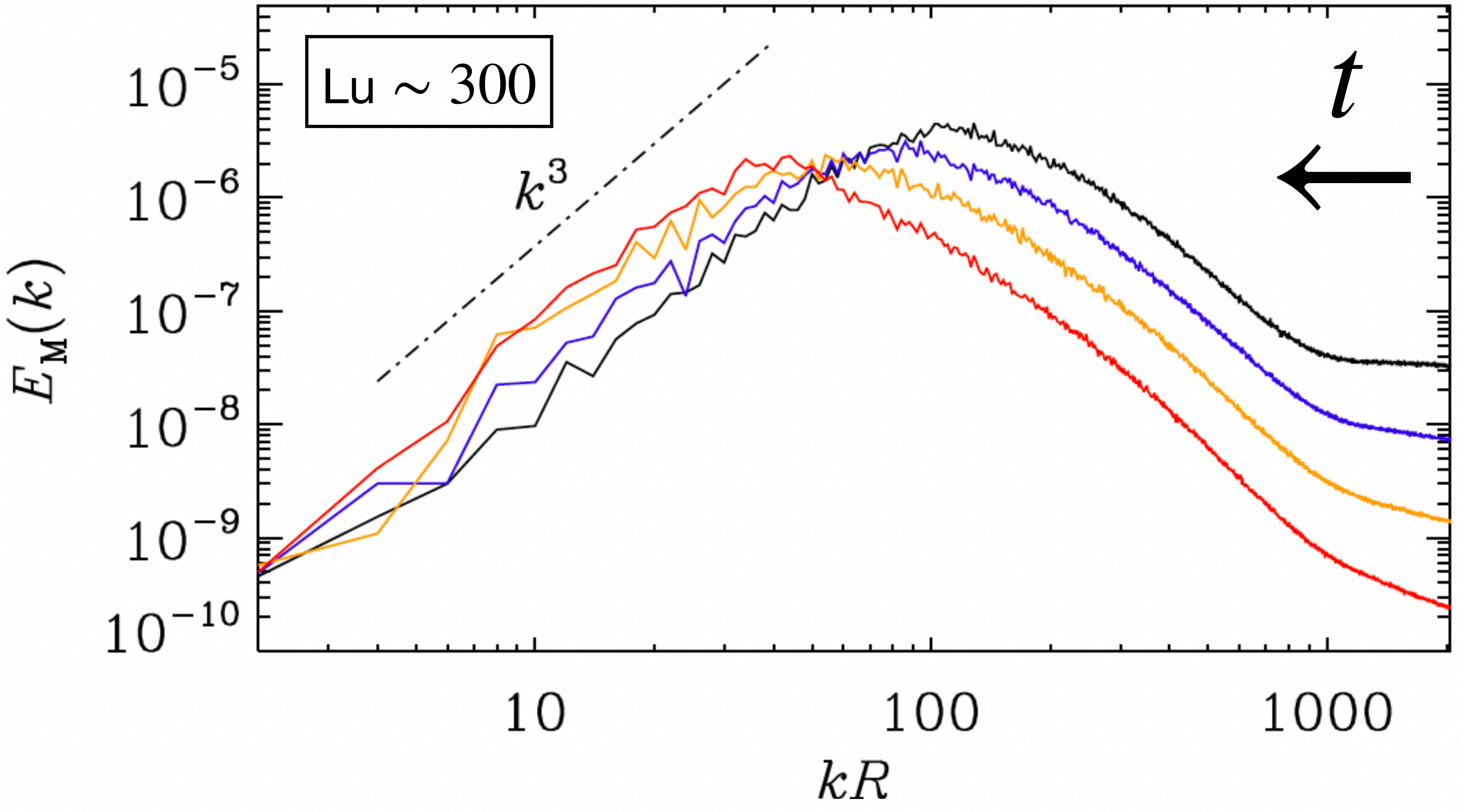
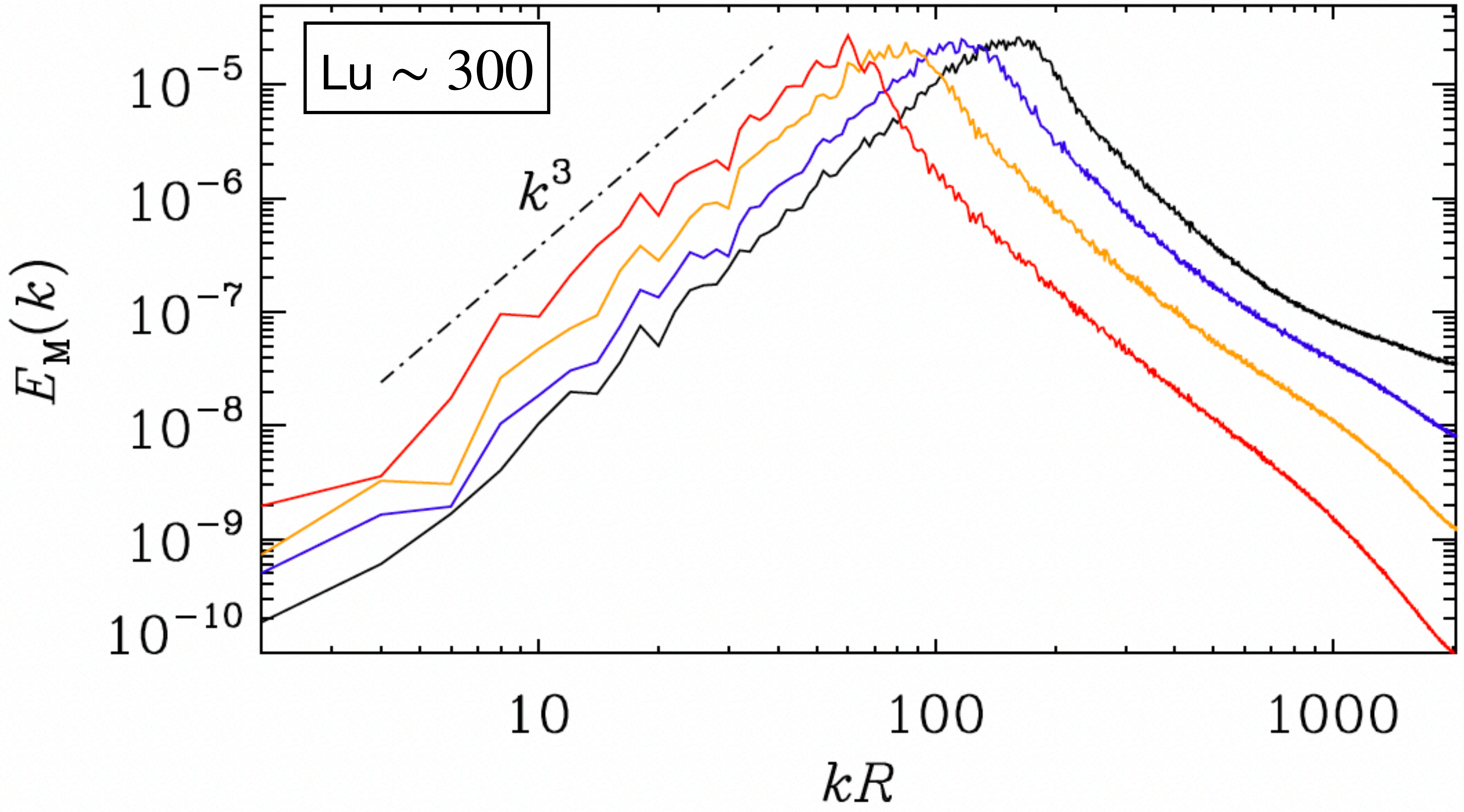
Inverse cascade: magnetic boundary conditions



Outer Boundary Conditions: Potential current-free

Inner Boundary Conditions: Perfect conductor



Outer Boundary Conditions: Periodic BC

Inner Boundary Conditions: Periodic BC

Typically used boundary conditions are causing further dissipation of the magnetic field.

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[Dehman & Brandenburg 2024 in prep.]

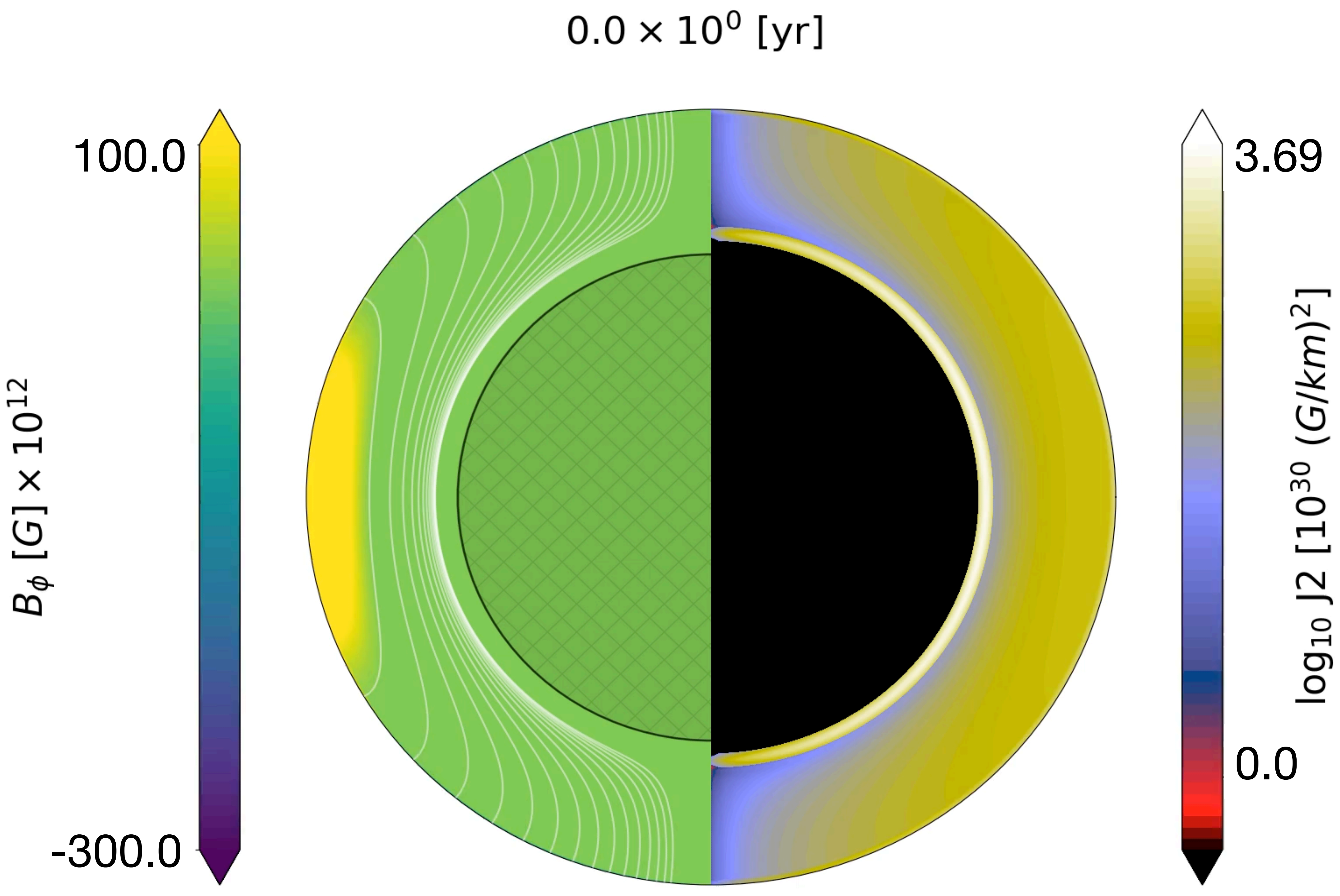
Left hemisphere:
Field lines: Poloidal magnetic field.
Colorbar: Toroidal magnetic field.

Right Hemisphere:
Electric current.

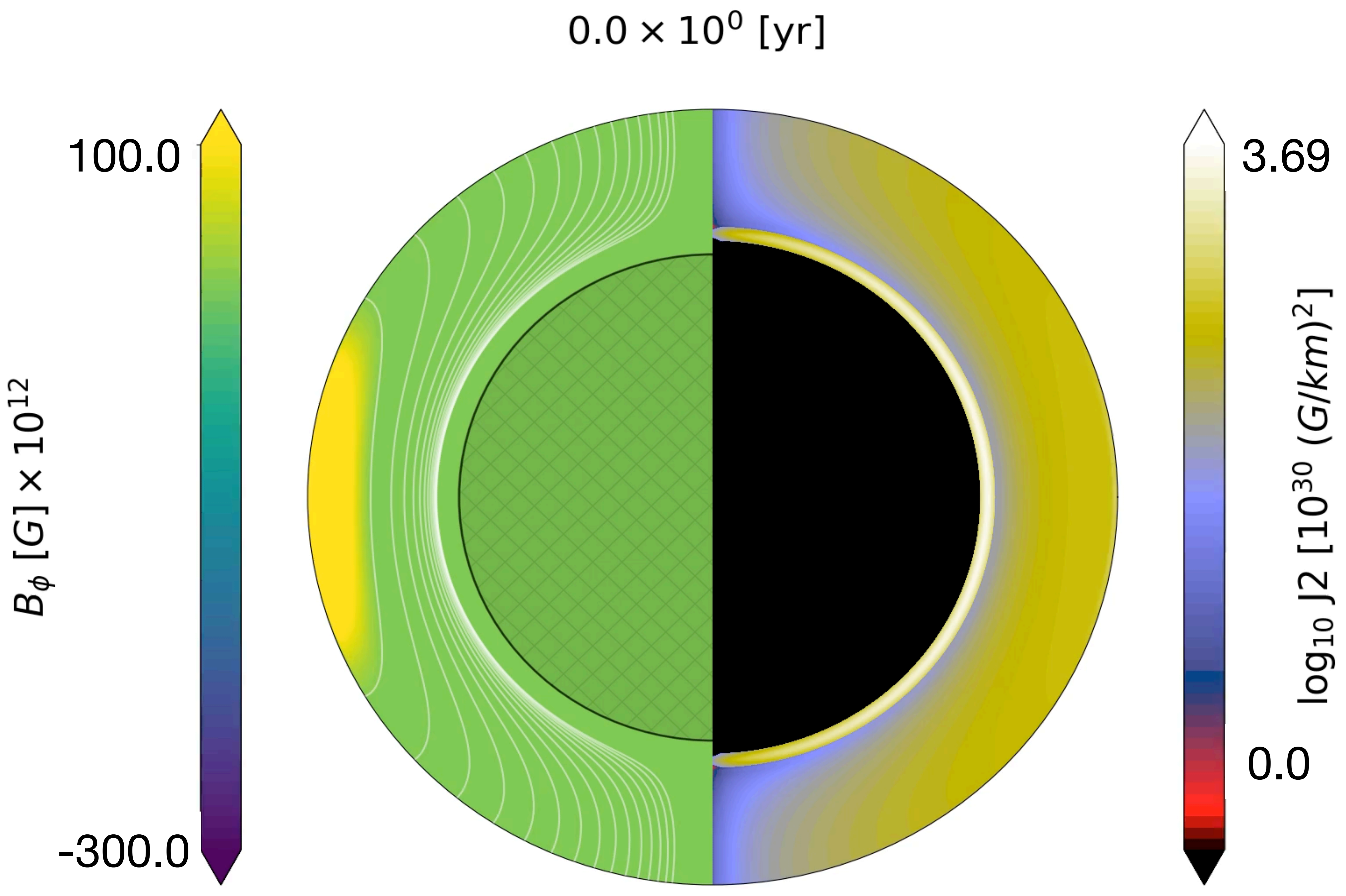
Force-free magnetosphere (2D)

—Physics informed neural network—

Force-free Magnetosphere



Vacuum Magnetosphere



Crust enlarged for visualisation purposes

The currents can flow in the magnetospheric $\left(\vec{\nabla} \times \vec{B} = \alpha \vec{B}\right)$.
Toroidal field does not vanish at the surface
$\mathcal{T}(\mathcal{P}) = s_1\mathcal{P} + s_2\mathcal{P}^2$

→

Crust-magnetosphere coupling affects the interior field evolution

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[Urban, Stefanou, Dehman & Pons 2023, MNRAS]

Summary & ongoing research

MATINS a new 3D code for magneto-thermal evolution in isolated neutron star crust.

"To be public soon"

Long-term evolution (10^5 yr) with a strong magnetic field $\sim 1e14$ G with R_m a few hundreds.

Proper treatments of microphysics, envelope models, axial singularity, field topology, temperature, etc.

Hall cascade, Inverse Hall cascade, outburst, etc.

Careful with boundary conditions…

A lot more can be explored !!

Magnetosphere in 3D using PINN

In collaboration with J. Urban, P. Stefanou & J. Pons

Chiral Magnetic Instability

In collaboration with Jose A. Pons & A. Brandenburg

Confronting observational data with 3D simulations

In collaboration with A. Marino, N. Rea & others

Axion field in neutron star crust

In collaboration with A. Gomez & J. Pons

Questions?

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